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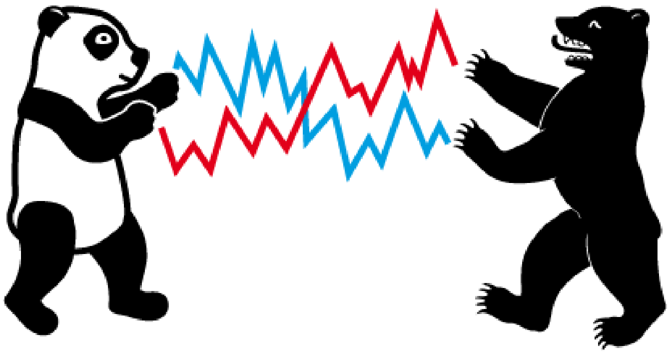

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Pricing Cryptocurrency options: the case of CRIX and Bitcoin

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Abstract

The CRIX (CRyptocurrency IndeX) has been constructed based on a number of cryptos and provides a high coverage of market liquidity, hu.berlin/crix. The crypto currency market is a new asset market and attracts a lot of investors recently. Surprisingly a market for contingent claims hat not been built up yet. A reason is certainly the lack of pricing tools that are based on solid financial econometric tools. Here a first step towards pricing of derivatives of this new asset class is presented. After a careful econometric pre-analysis we motivate an affine jump diffusion model, i.e., the SVCJ (Stochastic Volatility with Correlated Jumps) model. We calibrate SVCJ by MCMC and obtain interpretable jump processes and then via simulation price options. The jumps present in the cryptocurrency fluctutations are an essential component. Concrete examples are given to establish an OCRIX exchange platform trading options on CRIX.

Key Words: CRyptocurrency IndeX, CRIX, Bitcoin,Cryptocurrency, SVCJ, Option pricing,OCRIX

JEL Codes: C32, C58, C52

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1 Introduction

The Cryptocurrency (CC) market, in particular Bitcoin, has been receiving a lot of attention recently. Bitcoin (BTC) as a constituent of CRIX is based on a decentralized network and blockchain technology. It has by its very construction a pre-programmed inelastic money supply with a limit of 21 million bitcoins, which is going to be achieved by today's prediction in 2140. For the investors who want to ride the CRIX or BTC wave, the issue of pricing a CC derivatives is certainly interesting. The first aim of this study is to characterize and investigate the econometric properties of the CC prices. The second aim is to employ the traced dynamics for a continuous time model that permits contingent claims on CCs. More precisely, we calibrate the dynamics of the CRIX log returns to a stochastic volatility model with jumps. The advantage of this econometric model is that we obtain interpretable jump locations and frequencies.

The BTC price continuously soars to a record historical high, up to 12th December 2017, nearly 17613 USD. Its value has risen more than 2000 percent from around 1000 USD at the start of the year 2017. Extremely high trading volume and volatility in the CC market have been observed. Recently Ripple (XRP) has gained momentum and for a short period of time has taken the place of the second most liquid CC before Ethereum (ETH). A list of the most traded CCs is presented in an appendix.

Market participants express serious concerns for such a bumpy ride since any ascent may be followed by dramatic drops along the way. Coinmarketcap.com records as of today over 1400 CCs, many of them arising from ICOs (Initial Coin Offerings). The aggregate market capitalization of all CCs in circulation recently touched an all-time high exceeding 170 billion USD.

A growing number of newly created CCs and a rocket-alike speed of acceleration in trading volumes make it necessary to analyze this phenomenon scientifically.

The world's largest online trading platform, IG group that is an established member of the FTSE 250 with a market capitalization of 2.1 billion GPB, suspended at some point the trade of some CCs due to high speculative risk patterns. A rising risk especially takes place in the OTC market that accommodates big volume traders. The OTC market makers incur the unavoidable risk position from the very unbalanced orders between buying and selling. Sometimes when trading platforms cannot net customers' trades against each other, some of them will act themselves as a market maker. All these scenarios indicate the emergence of a counterparty risk, which endangers the functioning of trading platforms and damages the stability of the CC markets.

Due to the hedge demand against high volatility in this emerging but certainly immature market, the CME (Chicago Mercantile Exchange) group, the world's leading and most diverse derivatives marketplace, just launched BTC futures, based on the CME CF Bitcoin Reference Rate (BRR), on 18th Dec. 2017. Its main competitor, CFE (Cboe Futures Exchange), also launched trading in Cboe bitcoin futures on 10th Dec. 2017. To accompany this demand scientifically, this paper dives into option pricing not only for BTC but also for CRIX, the benchmark index for CCs. A correctly priced CC option provides the market with leverage, transparency, price discovery and risk transfer capacities. From the regulatory point of view, the ultimate goal is to reduce the risk of underlying market through introducing the corresponding derivative market as an alternative marketplace for speculators and arbitragers.

CRIX is designed to represent the market performance of leading CCs. It is based on a Laspeyre mechanism and it may be less volatile than most of its constituents. It is well understood that BTC is a highly volatile asset and this is consequently true also for CRIX. Ciaian et al. (2016) analyzes traditional determinants of price formation (demand and supply) together with specific factors of digital currencies: demand factors, such as attractiveness for investment, act as the keys for determining BTC price changes. BTC is also positively affected by an increase in information, as it creates a required awareness among users Bouoiyour et al. (2016). In summary, these findings conclude that despite being a highly speculative asset, speculation in this case is not fully undesirable.

Even if speculations indeed create price volatility and might lead to a bubble, BTC brings desirable liquidity into the market. Scaillet et al. (2018) states that new information (irrespective of fundamental or speculative nature) brings large volatility. However, comparing these findings one needs to be skeptical relative to the illiquidity of market and the absence of authorities. It should also be kept in mind that a large proportion of about 30% of the BTC universe is held by solely 112 people relative to 15 million BTC owners. The BTC market is extremely sensitive to large trading volumes and big swings. These results point to illiquidity and aggressive trades as key factors, creating jumps in the CC market.

A financial econometric analysis based on CRIX and BTC motivates a jump diffusion model. The data sample is from 01/08/2014 to 29/09/2017 and is downloaded from crix.berlin. We first consider standard time series models, as in Chen et al. (2017), including ARIMA, GARCH and EGARCH. It is discovered via residual analysis that the heavy tails of CRIX and apparent volatility clustering are not properly reflected. A swift move to continous time pricing models is therefore not justified! Given the numerous interventions and news driven shocks on the CC markets a jump component may help to integrate such events in a proper pricing model. More precisely, a diffusion associated with Stochastic Volatility and possibly Correlated Jumps (SVCJ) is motivated by these thoughts. In the SVCJ dynamics one may price CC derivatives. The SVCJ approach allows for correlated jumps in both returns and volatility, and enables us to capture the exogenously created extreme returns. This jump diffusion model is calibrated using MCMC. In summary, one achieves better estimations and calibrations compared to the models without jumps.

All calculations based on MATLAB quantlets can be found on www.quantlet.de. This platform is Github based and uses the display technique as described in Borke and Härdle (2018). The paper is organized as follows. Section 2 presents results on econometric analysis of BTC and CRIX. Section 3 discusses the SVCJ model, compares with diffusions without jump and our pricing formula. It also presents the technique of how to put a price on a contingent claim on CCs. Section 4 concludes the study.

2 The dynamics of CRIX and BTC

The CRyptocurrency IndeX CRIX developed by Härdle and Trimborn (2015) provides a market measure which consists of a selection of representative cryptos. Through the exceptional channel of an ICO, a CC startup can bypass the rigorous and regulated capital-raising process required by venture capitalists or banks. The appendix lists the top 10 constituents used to construct the CRIX index. The mechanism of selecting CRIX constituents is explained in Trimborn and Härdle (2018).

Figure 1 shows CRIX from 01/08/2014 to 29/09/2017. One observes that CRIX dropped substantially in mid 2015, perhaps as a result of a loss of interest in CCs. After a few months moving up and down, CRIX was, however, sloping up till beginning of 2017 as a stable period for CC markets. A dramatic soar was observed after March of 2017 due to a widespread interest in CCs. The subsequent drop in June was caused by a sequence of political interventions. The Chinese government has decided to force strict limits, temporary halts and even ban the CCs activities. The Chinese government announced a ban on ICOs, effectively shutting the financing door for many local startups. Recently the South Korean government announced similar measures to protect the small scale investors from possible total losses.

Figure 1 (left panel) indicates that neither CRIX nor BTC behaves like conventional stock prices, one records extremely high volatility and scattered spikes. Clearly, they are far from being stationary, the differentiation and detrending, or change point detection are required. Looking at their ACF a natural approach is to start with an ARIMA(p, d, q) model.

$$a(L)\Delta y_t = b_L \varepsilon_t \tag{1}$$

where y_t is the variable of interest, $\Delta y_t = y_t - y_{t-1}$, L is the lag operator and $\varepsilon_t \sim \mathrm{N}(0, \sigma^2)$.

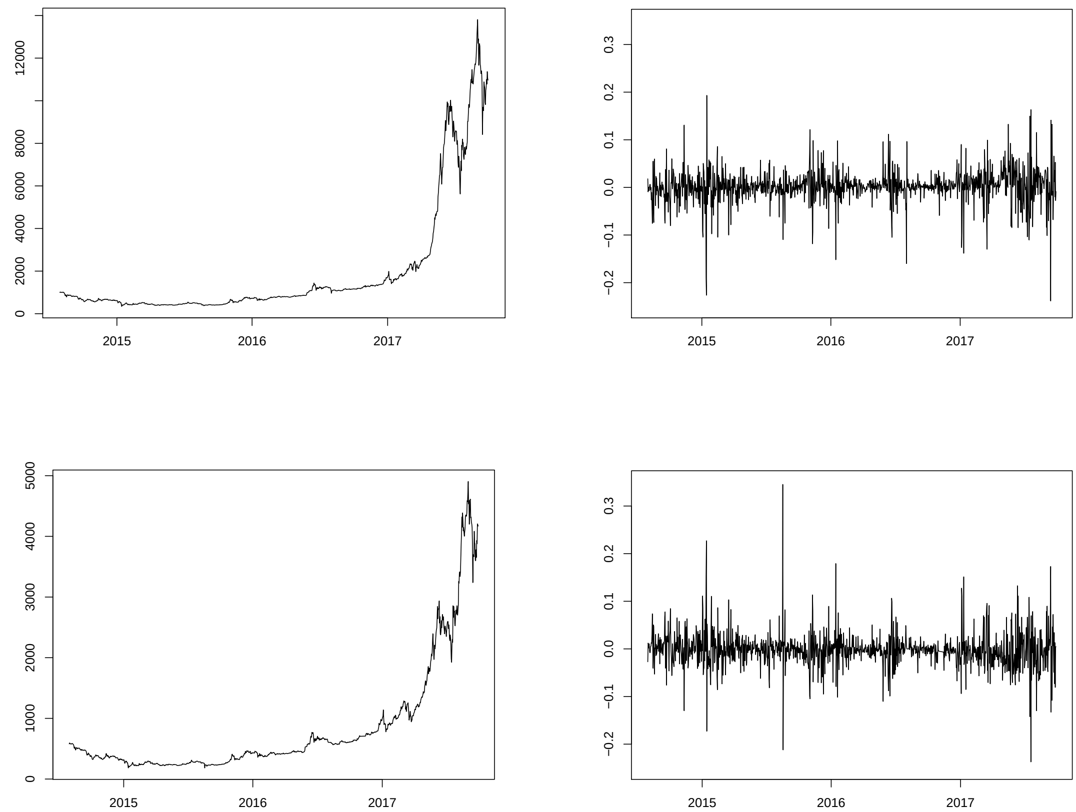

Figure 1: CRIX Daily Price and Return (upper panel) and BTCs Price and Return (lower panel) from Aug. 1st, 2014 to Sep. 29th, 2017

	CRIX		Bitcoin	
Coefficients	Estimate	Standard deviation	Estimate	Standard deviation
intercept c	0.002	0.001	-0.002	0.001
a_1	-0.819	0.188	-0.521	0.159
a_2	-0.791	0.112	-0.747	0.160
b_1	0.828	0.207	0.467	0.168
b_2	0.746	0.127	0.700	0.176
Log lik	2243.360	.	2139.340	.

Table 1: Estimation result of ARIMA(2,0,2) model

Model selection criteria like AIC or BIC indicates that the ARIMA$(2, 0, 2)$ is the model of choice. Similarly, BTC shares these features. The significant negative signs in a_1 and a_2 indicate an overreaction, that is, a promising positive return today leads to a return reversal in the following two days, or the other way around. It shows that the CC markets have a tendency to overreact good or bad news, and this overreaction can be corrected in the following two days. An ARIMA model for the CC assets, therefore, suggests a predictability due to an "overreaction".

The Ljung-Box test confirms that there is no serial dependence in the residuals based on the ARIMA$(2, 0, 2)$ specification, the details of these numerical computations are available in the quantlets used. Note that the squared residuals carry incremental information which are addressed in the following GARCH analysis.

2.1 GARCH Model

The GARCH model reflects the changes in the conditional volatility of the underlying asset in a parsimonious way. Duan (1995) develops a GARCH option pricing model in the context of the continuously compounded GARCH return process. A similar effort can be found in Heston and Nandi (2000). In Chen et al. (2017), GARCH and variants like t-GARCH, EGARCH have been reported, and they are seen to be nicely fitting the dynamics of CRIX but still could not handle the extreme tails in the residual distribution.

Let us start with a GARCH model for characterizing the conditional variance process of CRIX and BTC, respectively. The ARIMA-t-GARCH model is proposed first, with t-distributed innovations used to capture fat tails:

$$\begin{aligned} a(L)\Delta y_t &= b_L \varepsilon_t & (2) \\ \varepsilon_t &= Z_t \sigma_t, \quad Z_t \sim t(\nu) \\ \sigma_t^2 &= \omega + \beta_1 \sigma_{t-1}^2 + \alpha_1 \varepsilon_{t-1}^2 & (3) \end{aligned}$$

where σ_t^2 represents the conditional variance of the process at time t, $t(\nu)$ refers to the zero-mean t distribution with ν degrees of freedom. The covariance stationarity constraint $\alpha_1 + \beta_1 < 1$ is imposed. As shown in Table 2, the β_1 estimates from CRIX and BTC indicate a persistence in the variance process, but their values are relatively smaller than those estimated from the stock index return (see Franke et al. (2015)). Typically, the persistence-of-volatility estimates are very near one, showing that conditional models for stock index returns are very close to being integrated. By comparison, BTC and CRIX place a relatively higher weight in the α_1 coefficient and relatively lower weight in the β_1 to imply a less smooth volatility process and striking disturbances from the innovation term. This may further imply that the innovation is not pure white noise and can be occasionally contaminated by the presence of jumps.

In addition to the property of leptokurtosis, the leverage effect is commonly observed in practice. According to a large body of literature, starting with Engle and Ng (1993), the leverage effect refers to an asymmetric volatility response given a negative or positive shock. The leverage effect is captured by the exponential GARCH (EGARCH) model by Nelson (1991),

$$\begin{aligned} \varepsilon_t &= Z_t \sigma_t \\ Z_t &\sim t(\nu) \\ \log(\sigma_t^2) &= \omega + \sum_{i=1}^{p} \beta_i \log(\sigma_{t-i}^2) + \sum_{j=1}^{q} g_j\left(Z_{t-j}\right) & (4) \end{aligned}$$

where $g_j\left(Z_t\right) = \alpha_j Z_t + \phi_j(|Z_{t-j}| - \mathsf{E}|Z_{t-j}|)$ with $j = 1, 2, \dots, q$. When $\phi_j = 0$, we have the logarithmic GARCH (LGARCH) model from Geweke (1986) and Pantula (1986). To ac-

Coefficients	Estimates	robust std	t value
CRIX			
ω	$4.93e-05$	$2.69e-05$	1.83*
α_1	$2.23e-01$	$4.28e-02$	5.45***
β_1	$7.76e-01$	$5.62e-02$	13.81***
ν	$3.10e+00$	$2.19e-01$	14.15***
Bitcoin			
ω	$3.92e-05$	$1.49e-05$	2.63***
α_1	$2.28e-01$	$4.46e-02$	5.12***
β_1	$7.70e-01$	$5.13e-02$	14.98***
ν	$3.64e+00$	$4.08e-01$	8.91***

Table 2: Estimated coefficients of t-GARCH(1,1) model

$*$ represents significant level of 10% and $***$ of 0.1%.
The robust version of standard errors (robust std) are based on the method of White (1982).

econ_tgarch

commodate the asymmetric relation between stock returns and volatility changes, the value of $g_j(Z_t)$ must be a function of both the magnitude and the sign of Z_t. Over the range of $0 < Z_t < \infty$, $g_j(Z_t)$ is linear in Z_t with slope $\alpha_j + \phi_j$, and over the range $-\infty < Z_t \leq 0$, $g_j(Z_t)$ is linear in Z_t with slope $\alpha_j - \phi_j$.

The estimation results based on the ARIMA(2,0,2)-t-EGARCH(1,1) model are reported in Table 3. The estimated α_1 is no longer significant, showing a vanished sign effect. However, a significant positive value of ϕ_1 indicates that the magnitude effect represented by $\phi_1(|Z_{t-1}| - \mathsf{E}|Z_{t-1}|)$ plays a bigger role in the innovation in $\log(\sigma_t^2)$.

We compare the model performances between two types of GARCH models through information criteria, and find that a t-EGARCH(1,1) model for both CRIX and BTC is suggested. The shape estimates (ν) in Table 2 and 3 together with the QQ plots indicate a fatter tail in the return distribution of CRIX than that of BTC. Some constituents in the CRIX may behave more extremely than BTC does. Note that, as shown in Figure 2, the QQ plots demonstrate a deviation from the student-t. Being equipped with these findings and taking into account the occasional interventions, we opt for the models with jumps for better characterizing the CC dynamics. The presence of jumps is indeed more likely in this decentralized, unregulated and illiquid market. Numerous political interventions also suggest the introduce of jumps.

Coefficients	Estimates	robust std	t value
CRIX			
ω	$4.93e-05$	$2.69e-05$	1.83*
α_1	$5.58e-02$	$4.34e-02$	1.34
β_1	$9.62e-01$	$1.38e-02$	69.43***
ϕ_1	$5.36e-01$	$1.39e-01$	3.85***
ν	$2.42e+00$	$2.42e-01$	10.02***
Bitcoin			
ω	$3.84e-05$	$1.47e-05$	2.61***
α_1	$1.05e-03$	$5.10e-02$	0.98
β_1	$9.52e-01$	$1.54e-02$	61.73***
ϕ_1	$4.16e-01$	$6.64e-02$	6.25***
ν	$3.26e+00$	$4.16e-01$	7.82***

Table 3: Estimated coefficients of t-EGARCH(1,1) model

$*$ represents significant level of 10% and $***$ of 0.1%.
The robust version of standard errors (robust std) are based on the method of White (1982).

econ_garch

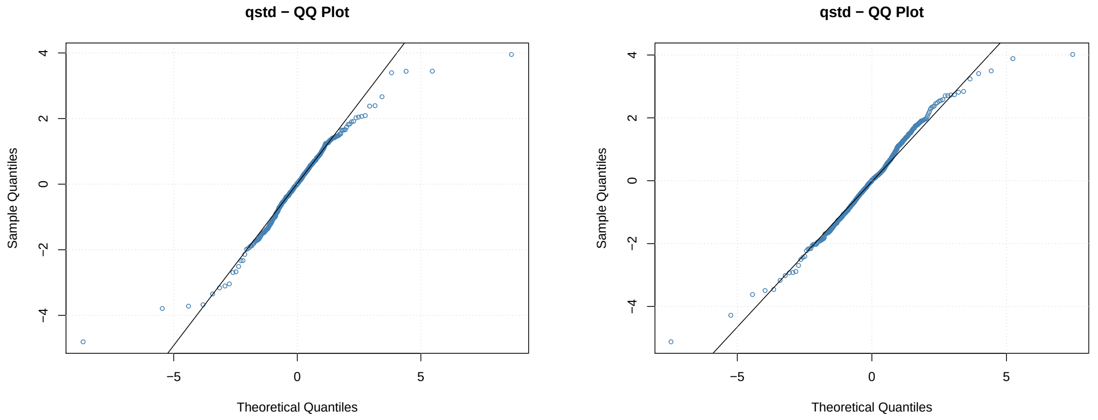

Figure 2: The QQ plot for CRIX (left) and Bitcoin (right) based on t-GARCH(1,1) model

3 Stochastic Volatility Model with Jumps

Although we have fitted a variety of financial econometrics model to the CRIX and BTC price, there still is evidence of non-Gaussianity and fat tails in the residuals. The political interventions followed by influental media comments in the past and the hype created sudden price moves motivates to consider more flexible and richer Stochastic Volatility (SV) models with jumps.

3.1 Models

In order to calibrate the CRIX dynamics with an SV part and Correlated Jumps (SVCJ) in return and volatility, we refer to the continuous time model of Duffie et al. (2000). The framework proposed in that paper indeed encompasses the standard jump diffusion and the SV with Jumps (SVJ) of Bates (1996). More precisely, let $\{S_t\}$ be the price process, $\{d\log S_t\}$ the log returns and $\{V_t\}$ be the volatility process, and the SVCJ dynamics are as follows:

$$d\log S_t = \mu dt + \sqrt{V_t} dW_t^{(S)} + Z_t^y dN_t \tag{5}$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma_V \sqrt{V_t} dW_t^{(V)} + Z_t^v dN_t \tag{6}$$

$$\mathsf{Cov}(dW_t^{(S)}, dW_t^{(V)}) = \rho dt \tag{7}$$

$$P(dN_t = 1) = \lambda dt \tag{8}$$

Like in the Cox-Ingersoll-Ross model, κ and θ are the mean reversion rate and mean reversion level respectively. $W^{(S)}$ and $W^{(V)}$ are two correlated standard Brownian motions with correlation denoted as ρ. N_t is a pure jump processes with a constant mean jump-arrival rate λ and the random jump sizes are Z_t^y and Z_t^v. Since the jump driving Poisson process is the same in both (5), (6), the jump sizes can be correlated. The random jump size Z_t^y conditional on Z_t^v, is assumed to have a Gaussian distribution with mean $\mu_y + \rho_j Z_t^y$ and standard deviation is set to be σ_y. The jump in volatility Z_t^v is assumed to follow an exponential distribution with mean μ_v:

$$Z_t^y | Z_t^v \sim \ N(\mu_y + \rho_j Z_t^v, \sigma_y^2); \quad Z_t^v \sim \ \text{Exp}(\mu_v). \tag{9}$$

The correlation ρ between the diffusion terms is introduced to capture the possible leverage effects between return and volatility. The media and hype interventions may be correlated as well, the correlation term ρ_j is taking care of that. The SV process $\sqrt{V}_t$ is modelled as a square root process. With no jumps in the volatility, the parameter θ is the long run mean of V_t and the process reverts to this level at a speed governed by the parameter κ. The parameter σ_V is referred to as the volatility of volatility and it measures the variance responsiveness to diffusive volatility shocks. In the absence of jumps the parameter μ measures the expected log-return.

This diffusion equation has been used in many financial time series applications, Eraker et al. (2003), Eraker (2004) and Broadie et al. (2007). It is a rich model since it covers SV and SVJ approaches as well. If we set $Z_t^v = 0$ in (9) such that jumps are only present in prices, we obtain the SVJ model of Bates (1996). Taking $\lambda = 0$, such that jumps are not present at all, the model reduces to the pure SV model originally proposed by Heston (1993). If we set $\kappa = \theta = \sigma_V = 0$ and define $Z_t^v = 0$ the model reduces to the pure jump diffusion introduced in Merton (1976).

3.2 Estimation: MCMC

An SVCJ model can be estimated by several methods. Calibration on observed option prices with the help of Fourier transformation has been advocated by Duffie et al. (2000)). There is however no market yet for options on CCs or the CRIX index. One therefore has to calibrate the SVCJ model using available numerical teechniques. Here we employ the Markov Chain Monte Carlo (MCMC) technique since it allows for a wide class of numerical fitting procedures that can be steered by variation of priors for example. As a caveat though one must keep in mind, that given that there are no options yet, one may not be able to reflect the correct market price of risk, Franke et al. (2015). The empirical calibration is based on the following Euler discretization,

$$Y_t = \mu + \sqrt{V_{t-1}}\varepsilon_t^y + Z_t^y J_t \quad (10)$$

$$V_t = \alpha + \beta V_{t-1} + \sigma_V \sqrt{V_{t-1}}\varepsilon_t^v + Z_t^v J_t \quad (11)$$

where $Y_{t+1} = \log(S_{t+1}/S_t)$ is the log return, $\alpha = \kappa\theta$, $\beta = 1 - \kappa$ and ε_t^y, ε_t^v are the $N(0,1)$ variables with correlation ρ. J_t is a Bernoulli random variable with $P(J_t = 1) = \lambda$ and the jump sizes Z_t^y and Z_t^v are distributed as specified in (9).

Now we give a brief description on how to calibrate SVCJ with MCMC, see also Johannes and Polson (2009), Tsay (2005), Asgharian and Bengtsson (2006) for more details. Define the parameter vector $\Theta = \{\mu, \mu_y, \sigma_y, \lambda, \alpha, \beta, \sigma_v, \rho, \rho_j, \mu_v\}$, Recall Y_t as the log-returns and define $X_t = \{V_t, Z_t^y, Z_t^v, J_t\}$ as the latent variance, jump sizes and jump. MCMC treats all components of both Θ and $X \stackrel{\text{def}}{=} \{X_t\}_{t=1,..,T}$ as random variables. The fundamental quantity is the joint pdf $p(\Theta, X|Y)$ of parameters and latent variables conditioned on data. Using Bayes formula

$$p(\Theta, X|Y) = p(Y|\Theta, X)p(X|\Theta)p(\Theta). \tag{12}$$

it can be decomposed into three factors: $p(Y|\Theta, X)$, the likelihood of the data, $p(X|\Theta)$ the prior of the latent variables conditioned on the parameters, and $p(\Theta)$ the prior of the parameters. The prior distribution $p(\Theta)$ has to be specified beforehand and is part of the model specification. In comfortable settings the posterior variation of the parameters given the data is robust with respect to the prior. We will touch this point again when we display our empirical results.

The posterior is typically not available in closed form and therefore simulation is used to obtain random draws from it. This is done by generating a sequence of draws, $\{\Theta^{(i)}, X_t^{(i)}\}_{i=1}^N$ which form a Markov chain whose equilibrium distribution equals the posterior distribution. The point estimates of parameters and latent variables are then taken from their sample means. We present our results for the following prior for the parameters: $\mu \sim N(0, 25)$, $(\alpha, \beta) \sim N(0_{21}, I_{22})$, $\sigma_2^V \sim IG(2.5, 0.1), \mu_y \sim N(0, 100), \sigma_2^y \sim IG(10, 40), \rho \sim U(1, 1), \rho_j \sim N(0, 0.5), \mu_V \sim IG(10, 20)$(Inverse Gaussian) and $\lambda \sim Be(2, 40)$ (Beta Distribution). We have varied the variance of the priors and found stable outcomes, ie. the reported mean of the posterior that is taken as an estimate of Θ is quite robust relative to changes in prior variance. A similar observation has been made by Asgharian and Nossman (2011) for example, and is chosen such that it represents a wide range of possible realistic estimates. With these choices the posterior for all parameters except σ_V and ρ are all conjugate (meaning that the posterior distribution is of the same type of the distribution as the prior but with different parameters). The posterior for J_t is a Bernoulli distribution. The jump sizes Z_t^y and Z_t^v follow a posterior normal distribution and a truncated normal distribution respectively. Hence, it is straightforward to obtain draws for the joint distribution of J_t, Z_t^y and Z_t^v . However, the posteriors for ρ, σ_2^V and V_t are non-standard distributions and must be sampled using the Metropolis-Hastings algorithm. We use the random walk method for ρ and V_t , and independence sampling for σ_V^2. For the estimation of posterior moments, we perform 5000 iteratations, and in order to reduce the impact of the starting values we allow for a burn-in for the first 1000 simulations.

SVCJ is known to be able to disentangle returns that are related to sudden unexpected jumps from large diffusive returns caused by periods of high volatility. For the CC situation that we consider here we are particularly interested in linking the latent historical jump times to news and known interventions. The estimates $\hat{J}_t \stackrel{\text{def}}{=} (1/N) \sum_{i=1}^{N} J_t^i$ (where N is the total number of iterations, i refers to each draw) indicate the posterior probability that there is a jump at time t, unlike the "true" vector of jump times, it will not be a vector of ones and zero. Following Johannes et al. (1999), we assert that a jump has occured on a specific date t if the estimated jump probability is sufficiently large; that is, greater than an approporiately chosen threshold value:

$$\tilde{J}_t = 1\{\hat{J}_t > \zeta\}, \quad t = 1, 2, ..., T \tag{13}$$

In our empirical study we choose ζ so that the number of inferred jump times divided by the number of observations is approximately equal to the estimate of λ.

The parameter estimates (mean and variance of the posterior) for the SVCJ, SVJ and SV model for CRIX and BTC are presented in Table 4 and 5, respectively. The estimate of μ is positive, ρ the correlation between return and volatility is significant and positive. This is remarkable and worth noting since it is different from a positive leverage effect that is observed consistently over a sequence of studies, see e.g. Eraker (2004). One reason for this might be that CRIX and

BTC are not informative but rather emotional and sentiment driven and are therefore different from conventional stock prices. The diagnosis that we can make here is more consistent with the "inverse leverage effect" that has been claimed for commodity markets, Schwartz and Trolled (2009). In other words, the"inverse leverage effect" (associated with a positive ρ) implies that increasing prices are associated with increasing volatility. Moreover, the estimates for the SV part of SVCJ is much less extreme than of the SVJ and the SV models. More precisely, the volatility of variance σ_v being substantially reduced from 0.017 (SV) to 0.011(SVJ) and 0.008 (SVCJ).

The mean of the jump size of the volatility μ_v is significant and positive. The jump intensity λ is also significant. The jump correlation ρ_j is negative and insignificant, paralleling results of Eraker et al. (2003) and Chernov et al. (2003) for stock price dynamics. This might be due to the fact that even with a long data history, jumps are rare events. Im summary the SVCJ model fits the data well by a smaller overall MSE.

Table 4: CRIX parameters for SVCJ, SVJ and SV, *** (**, *)means significance at 1% level(5%, 10%).

	$SVCJ$		SVJ		SV	
	$Mean$	$Std.Dev$	$Mean$	$Std.Dev$	$Mean$	$Std.Dev$
μ	0.042***	0.006	0.044***	0.009	0.023***	0.009
μ_y	-0.049	0.371	-0.515	0.303	-	-
σ_y	2.061***	0.432	2.851***	0.767	-	-
λ	0.051***	0.007	0.035***	0.009	-	-
α	0.010***	0.001	0.026	0.019	0,010***	0.001
β	-0.19***	0.009	-0.24***	0.073	-0.04***	0.009
ρ	0.275***	0.069	0.214**	0.102	0.003***	0.068
σ_v	0.007***	0.001	0.016*	0.009	0.018***	0.002
ρ_j	-0.210	0.364	-	-	-	-
μ_v	0.709***	0.089	-	-	-	-
MSE	0.673	-	0.702	-	0.736	-

Figure 3 shows the estimated jump in returns (first row), in volatility (middle row) together with the estimated volatility (last row) for CRIX (left column) and BTC (right column). One sees that jumps occur frequently both for the return and volatility for both CRIX and BTC. Apparently, the jumps in volatility process is much larger and more frequent than in the returns for both CRIX and BTC. It seems that for BTC, jump sizes are smaller and occur less frequently both for returns and volatility.

Table 5: BTC parameters for SVCJ, SVJ and SV model, *** (**, *)means significance at 1% level(5%, 10%).

	$SVCJ$		SVJ		SV	
	$Mean$	$Std.Dev$	$Mean$	$Std.Dev$	$Mean$	$Std.Dev$
μ	0.041 ***	0.010	0.029***	0.009	0.030***	0.008
μ_y	-0.084	0.385	-0.562	0.366	-	-
σ_y	2.155***	0.517	2.685***	0.595	-	-
λ	0.041***	0.008	0.033***	0.007	-	-
α	0.010***	0.001	0.010	0.002	0.009***	0.002
β	-0.132***	0.009	-0.116***	0.011	-0.033***	0.010
ρ	0.407***	0.089	0.321***	0.049	0.169***	0.052
σ_v	0.008***	0.001	0.011***	0.002	0.017***	0.002
ρ_j	-0.573	0.642	-	-	-	-
μ_v	0.620***	0.099	-	-	-	-
MSE	0.735	-	0.757	-	0.763	-

Figure 4 presents the in-sample fitted volatility processes for SVCJ and SVJ respectively. The maximum volatility level for both CRIX and BTC were found in October and November, 2017 right before the recent dramatic increase of CC prices. BTC has a smaller estimated volatility. Moreover, it is not hard to see that both SVJ and SVCJ models lead to a similar overall pattern of the volatility process though the SVCJ model produces sharper peaks for CRIX and BTC.

A useful model diagnostic is to examine the residuals obtained from the discrete model,

$$\varepsilon_t^y = \frac{Y_t - \mu - Z_t^y J_t}{\sqrt{V_{t-1}}} \tag{14}$$

Once these residuals are calculated based on the estimated parameters, they should, according to (9), be approximately standard normally distributed. Figures 5 and 6 show the QQ plots of the residuals coming from the fitting of different models. From these diagnostics, it is evident that the GARCH and even the SV models are misspecified. For the SVJ and SVCJ models the QQ plot diagnostics are substantially improved, however, it is apparent that SVCJ is the preferred choice.

Finally Figure 7 graphs the 5%, 25%, 75% and 95% percentiles of the 5000 simulated paths for each horizon up to 30 days for the different models considered here. This can be considered as an interval forecast of BTC and CRIX. There is no big difference between the results. ARIMA has the smallest average width of among all models. SCVJ, SVJ, SV produce similar forecast

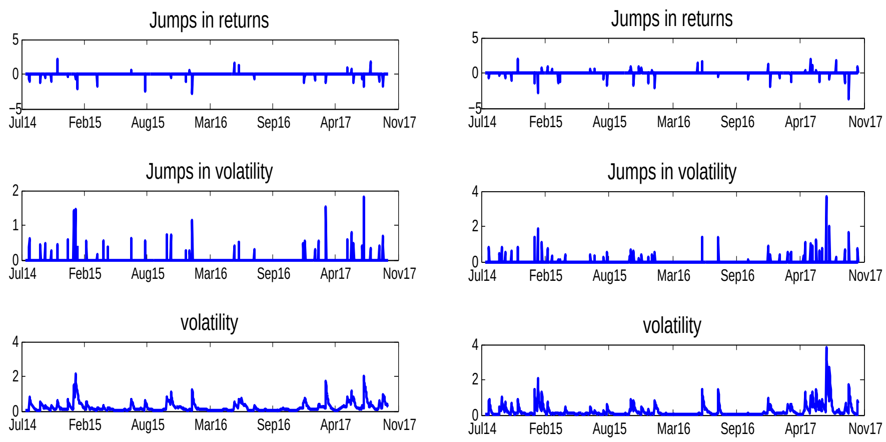

Figure 3: Jumps in returns and volatility from the SVCJ model for Crix(left panel) and Bitcoin(right panel).

econ_SVCJ

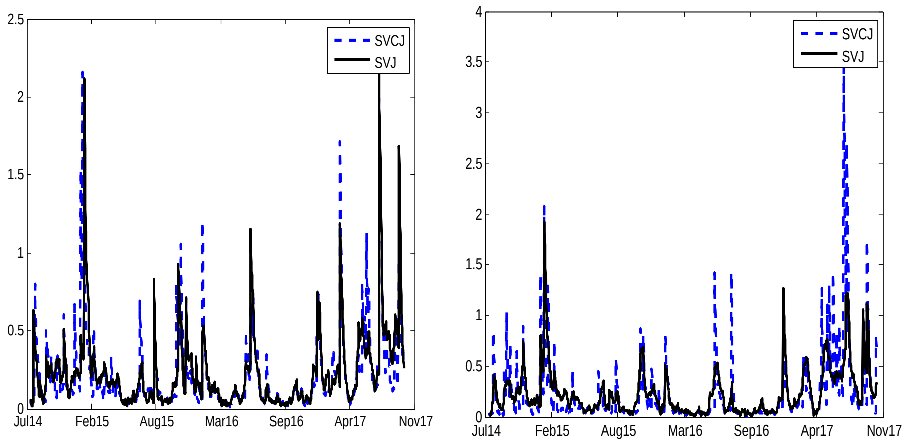

Figure 4: Estimated volatility from SVCJ and SVJ models for Crix(left panel) and Bitcoin(right panel).

econ_SVCJ

results, and SVCJ results to a wider interval in some forecast horizons. SVCJ, SVJ, SV produce similar forecast results at the 25th and 75th level. However, at the 5% (95%) percentile, the SV and SVCJ models present more difference in term of forecast. This is the same when we look at the BTC price forecast at the 95% percentile, both SV and SVJ models over estimate the underlying price while at the 5% percentile, SV (SVJ) over (under) estimate the BTC prices.

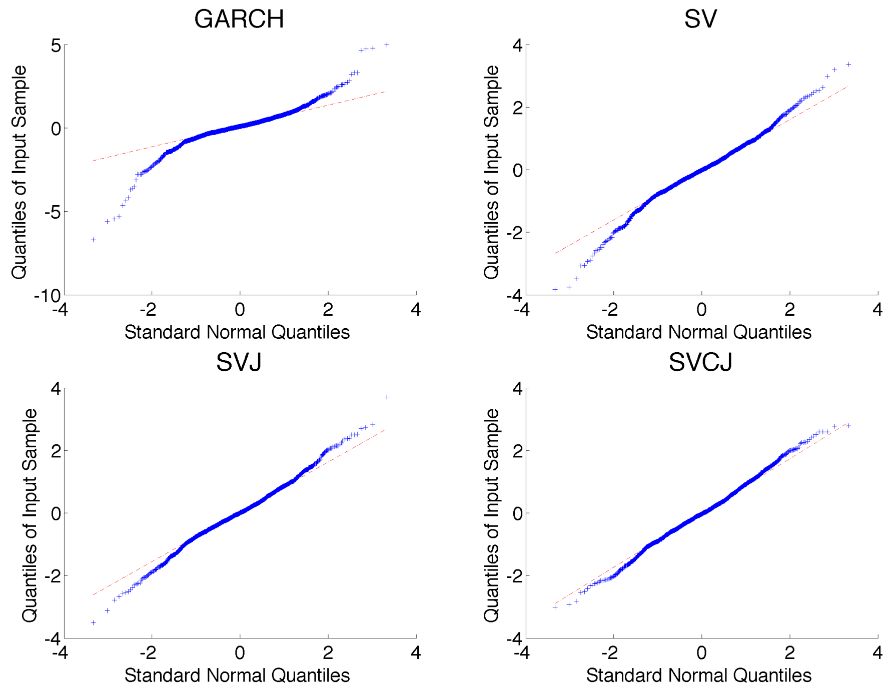

Figure 5: Normal probability plots for SVCJ, SVJ, SV models for Crix.

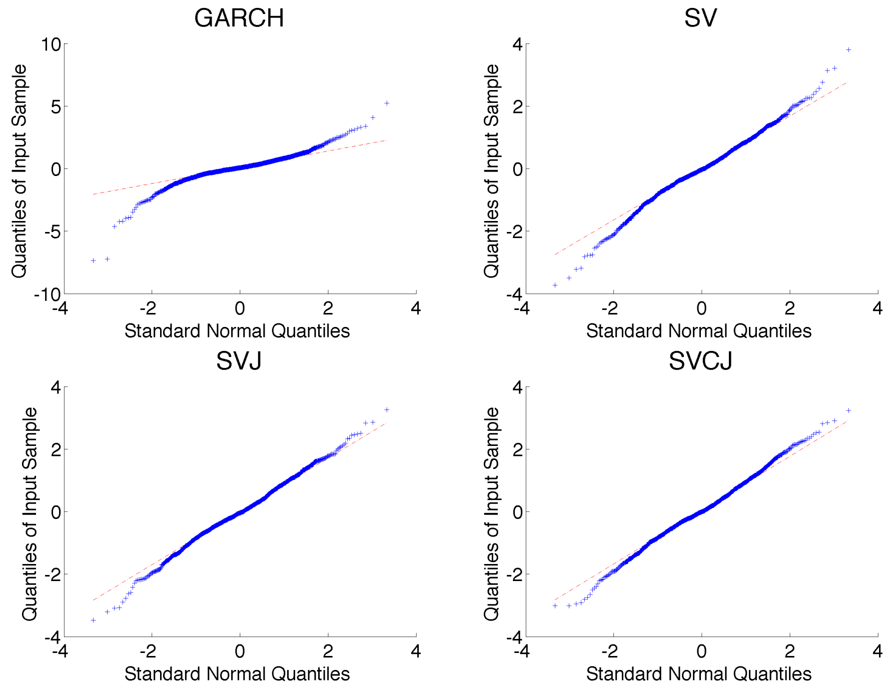

Figure 6: Normal probability plots for SVCJ, SVJ, SV models for Bitcoin.

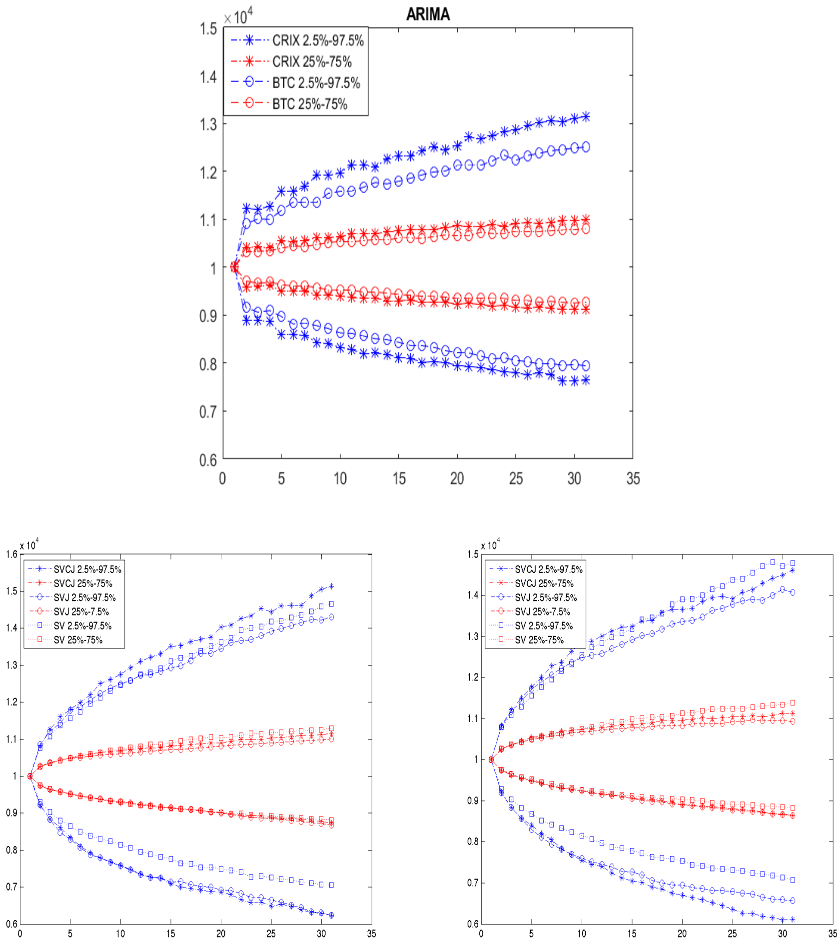

Figure 7: Confidence intervals of simulated observations for different models for ARIMA for both Crix and Bitcoin (upper panel), Crix (lower left) and Bitcoin (lower right)

econ_SVCJ

3.3 Option pricing

After we fixed the SVCJ parameters, we advance with a numerical technique called Crude Monte Carlo to approximate the CRIX option prices. Derivative securities such as futures and options are priced under a probability measure Q commonly referred to as the "risk neutral" or martingale measure. Since our purpose is to explore the impact of model choice on option values we follow Eraker et al. (2003) and set the risk premia to be 0. This is disputable, but for the lack of existence of the officially traded options a justifiable path to pricing OCRIX contingent claims.

Suppose we have an option with pay-off at time of maturity T as $C(T)$, and typically for call option $C(T) = (S_T - K)^+$. The price of this option at time t is denoted as

$$\mathsf{E}_Q[\exp\{-r(T-t)\}C(T)|\mathcal{F}_t]$$

The crude Monte Carlo is done for 10000 iterations to approximate the option price. Moneyness for strike K at t is defined to be K/S_t. The pricing formulae are functions of moneyness and time to maturity $\tau = (T - t)$ and are not available in closed form.

The fitted call option prices on time 20170717 using SVCJ are presented in Table 6 for CRIX assuming a CRIX level of $S_t = 6500$) and Table 7 for BTC assuming BTC price $S_t = 2250$) We see that for example, a call on BTC with the strike $K = 1250$ and time to maturity 60 days would be traded at 1157.95 at time 20170717. For the prices produced by SV and SVJ model, we refer to Table 9, 10, 11 and 12 in the Appendix.

Figure 8 shows the estimated price of call options with respect to moneyness assuming 30 days time to maturity) and assuming an ATM situation at 20170717. It can be seen that the price drops with increase of moneyness and it increases gradually with time to maturity. To futher understand how does the price changes with respect to time to maturity and moneyness over time for different models, we show in Figure 10 and Figure 11, the one-dimensional contour plot of the option price on 20170717 estimated from SVCJ, SVJ and SV. For both CRIX and

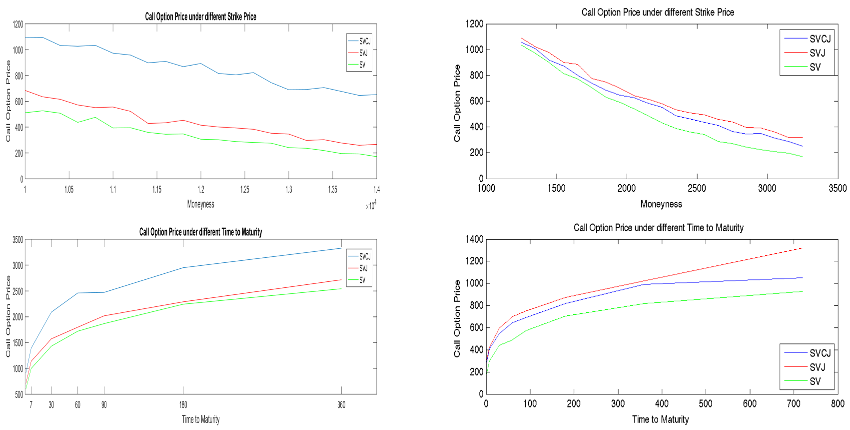

Figure 8: Call option prices C against K and time to maturity, 20170717. Crix(left panel) and Bitcoin(right panel)

econ_SVCJ

BTC, we can see from the contour plot that the relationship between the price and the time to maturity or moneyness varies over time for all three models. However, the SVCJ model has the most volatile pattern among the three. In particular, for BTC we see a drastic change in the contour structure on e.g. 20170715, as the price suddenly drops from 2232.65 USD on 20170714 to 1993.26 USD on 20170715.

Figure 12 and 13 diplay the price difference between the estimated option price from the SVJ and the SVCJ model with respect to moneyness and time to maturity. It is not hard to see that the pattern is similar to the fitted volatility function as shown in Figure 9. Therefore the price difference between SVJ and SVCJ model is mainly contributed by the jump in the volatility process, which reflects the necessity of adopting a SVCJ model in practice.

4 Conclusion

"The Internet is among the few things that humans have built that they do not truly understand" Schmidt and Cohen (2017). Well, if people do not understand the Internet, how are they expected to understand CCs? There is an enormous amount of misinformation that created such misunderstanding though. CCs are simply new asset classes, that may be thought of as cur-

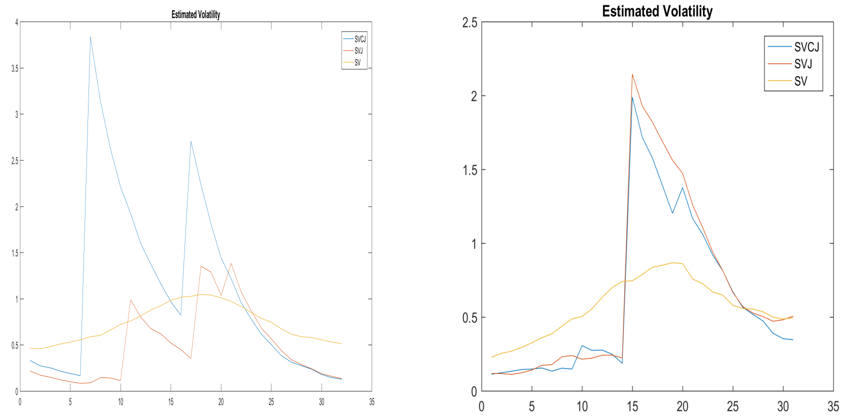

Figure 9: Estimated volatility functions, 201707. Crix(left panel) and Bitcoin(right panel).
econ_SVCJ

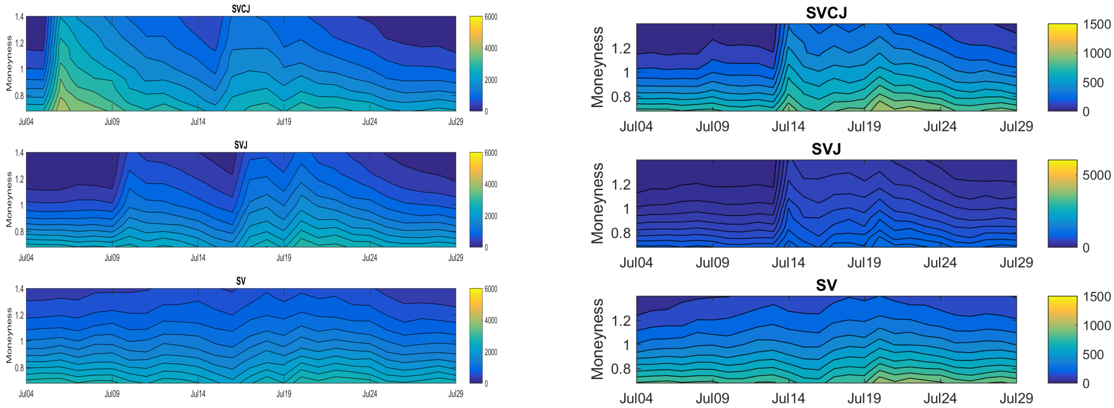

Figure 10: Call option price against moneyness, 201707. Crix(left panel) and Bitcoin(right panel).

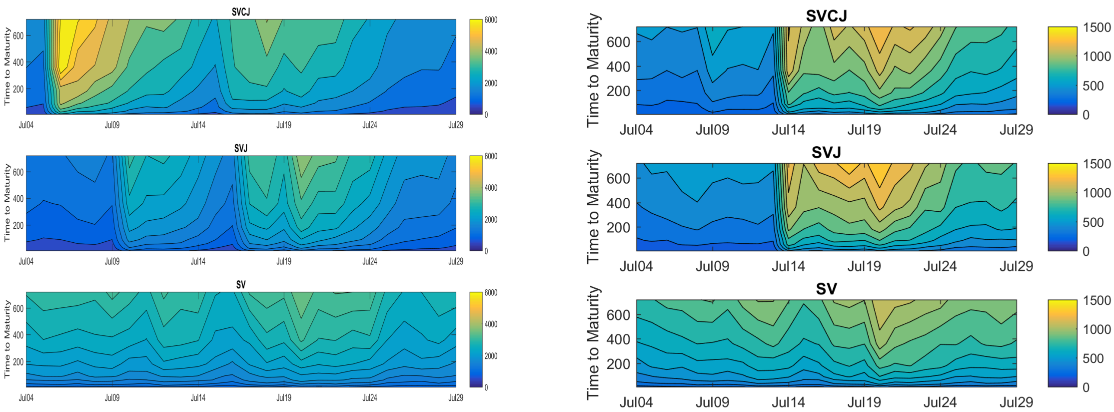

Figure 11: Call option price against time to maturity, 201707. CRIX(left panel) and BTC(right panel).

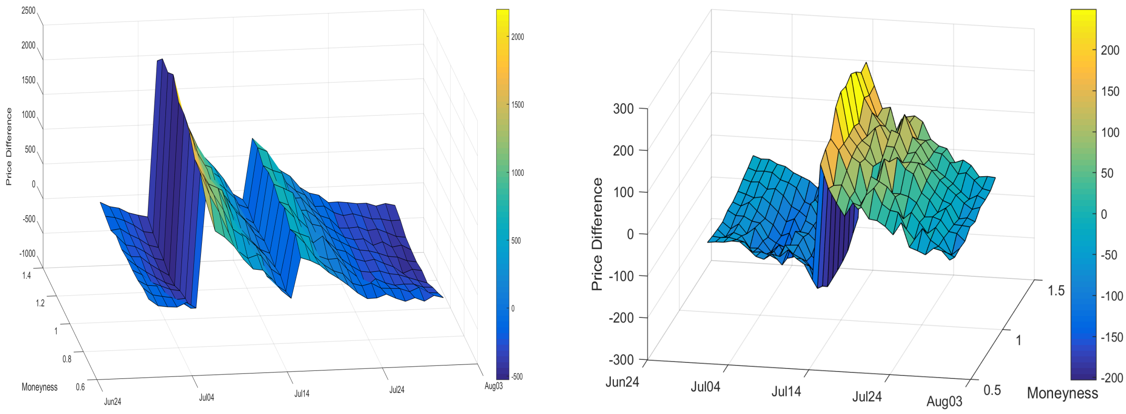

Figure 12: Price difference between the SVJ and SVCJ model plotted against time moneyness, 201707. CRIX(left panel) and BTC(right panel).

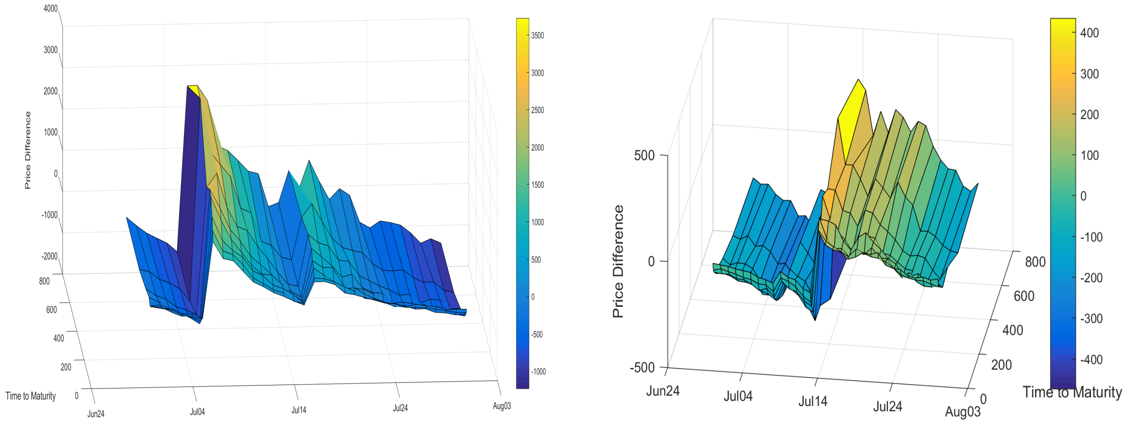

Figure 13: Price difference between the SVJ and SVCJ model plotted against time to maturity. CRIX (left panel) and BTC (right panel).

Table 6: Option price for CRIX call option at different time to maturity τ and strike prices K for SVCJ. 20170717

$K\backslash\tau$	1.00	7.00	30.00	60.00	90.00	180.00	360.00	720.00
5500.00	1700.70	1971.69	2376.41	2661.03	2836.39	3359.68	3670.35	4175.19
5600.00	1622.22	1861.35	2346.51	2745.06	2836.35	3188.52	3583.43	4473.40
5700.00	1509.72	1865.38	2310.64	2735.19	2773.45	3185.18	3381.22	3851.61
5800.00	1517.11	1797.53	2284.87	2610.68	2841.49	3026.03	3858.18	4166.42
5900.00	1451.34	1710.80	2190.93	2426.56	2764.44	2978.83	3532.85	4373.64
6000.00	1372.53	1649.63	2149.38	2423.62	2645.95	3056.08	3728.81	3857.57
6100.00	1325.74	1610.69	2062.01	2471.14	2647.20	3049.92	3673.39	4078.56
6200.00	1286.20	1649.02	2078.84	2384.83	2710.22	2984.17	3065.60	3787.80
6300.00	1257.31	1590.43	2066.53	2319.82	2712.00	2761.42	3566.18	4778.50
6400.00	1194.82	1584.19	2003.73	2350.16	2586.18	2825.99	3320.98	3924.42
6500.00	1153.04	1497.01	1913.27	2303.17	2605.96	2975.90	3378.30	4161.48
6600.00	1099.42	1477.81	1885.04	2252.67	2502.56	3058.84	3305.54	3944.97
6700.00	1020.15	1437.87	1872.02	2173.94	2505.58	2747.43	3453.51	3852.04
6800.00	974.45	1367.32	1908.37	2174.41	2372.43	2839.14	3616.94	4051.21
6900.00	932.90	1311.90	1858.10	2117.97	2488.77	2998.27	3070.65	3791.22
7000.00	885.79	1296.37	1853.47	2200.17	2388.94	2674.59	3591.75	3617.75
7100.00	823.41	1258.83	1742.18	2096.70	2329.85	2693.00	3665.56	4025.92
7200.00	799.13	1259.14	1730.04	2128.48	2412.37	3013.89	3063.93	4102.05
7300.00	765.20	1251.43	1624.24	2118.81	2331.00	2730.96	3145.85	3462.55
7400.00	778.72	1176.72	1677.32	2034.31	2248.58	2709.87	3249.47	4388.28
7500.00	685.19	1135.72	1701.65	2083.99	2375.38	2787.90	3114.68	3619.57

Table 7: Option price for Bitcoin call option at different time to maturity τ and strike prices K for SVCJ. 20170717

$K\backslash\tau$	1.00	7.00	30.00	60.00	90.00	180.00	360.00	720.00
1250.00	1069.18	1017.81	1099.87	1125.90	1157.95	1248.98	1361.04	1365.96
1350.00	959.02	959.02	1006.02	1066.67	1094.08	1224.48	1302.60	1316.03
1450.00	885.20	860.15	929.32	995.45	1046.89	1099.35	1258.83	1438.90
1550.00	802.38	791.34	901.27	950.34	1015.76	1114.94	1192.24	1332.08
1650.00	707.97	739.10	825.07	882.17	902.32	1062.17	1175.59	1282.36
1750.00	625.86	678.22	786.88	856.72	896.56	962.79	1192.61	1338.49
1850.00	552.26	618.94	697.11	785.62	862.83	897.74	1110.36	1289.51
1950.00	502.28	545.58	663.47	740.32	819.72	903.60	1052.09	1229.45
2050.00	425.46	511.28	629.14	741.65	772.51	905.30	1027.76	1193.43
2150.00	358.30	460.57	597.44	683.55	740.64	870.66	1036.76	1164.23
2250.00	302.88	408.62	543.02	633.31	720.57	872.42	938.68	1051.71
2350.00	265.91	378.10	492.86	594.01	651.03	783.37	887.62	1064.33
2450.00	211.26	347.79	470.85	580.30	657.43	761.39	940.90	1085.75
2550.00	193.69	304.13	437.06	547.15	608.36	766.19	914.62	1101.72
2650.00	156.38	266.64	421.86	518.27	571.42	719.92	827.17	992.20
2750.00	136.24	247.38	397.92	484.70	556.31	651.86	863.10	1066.75
2850.00	135.28	228.47	345.42	465.75	541.61	672.76	788.25	955.97
2950.00	100.02	202.57	341.11	413.75	488.15	627.52	780.53	917.27
3050.00	103.45	179.93	313.83	424.23	496.05	619.88	758.99	911.33
3150.00	82.59	162.72	290.90	371.20	450.85	593.10	752.88	888.89
3250.00	72.93	140.40	273.97	358.26	442.91	571.96	726.49	933.57

rencies in a non physical form or of commodities or if one prints out the CC account that one holds as fiat money on paper. As a currency they are all trust based to enable transactions and establishment of markets.

As an asset class a secondary option market is a natural enrichment for the control of volatility, for price discoveries and for enhanced performance of forecast. In this paper we analyse the benchmark index CRIX (crix.berlin) for its econometric dynamics. We find that standard financial econemetric models based on GARCH and its variants are not able to reflect the deep dark tail situations in CRIX and e.g. BTC for the reason that these models do not reflect the jump presence. We therefore calibrate the dynamics of the CRIX and BTC log returns to a stochastic volatility model with jumps. The advantage of this econometric model is that we obtain interpretable jump locations and frequencies. It also allows for pricing of contingent claims.

The SVCJ model is estimated via MCMC and shows stable results over a variety of priors. We present an almost perfect fit in a distributional sense. The QQ plots are precisely confirming the Gaussian innovations that drive the continuous time diffusions process with stochastic volatility. We present a tableau of option prices for different time to maturity and moneyness. The calculation are done via Monte Carlo and under the assumption of market price of risk to be equal to zero. This has to be done since there is no established option market yet. We believe that this research helps in establishing an option market for CCs in the near future.

5 Appendix:List of CCs

The Research Data Center RDC hu-berlin/rdc supported by IRTG 1792 provides an access to the dataset. At time of writing, BTCs market capitalization as a percentage of CRIX total market capitalization is 41%. Currently, the CRIX consists of 75 index members. This number was found with statistical methods, see for more details in http://crix.hu-berlin.de. Here we describe the top 10 coins collected in the CRIX.

No.	Cryptos	Symbol	Description
1	Bitcoin	BTC	BTC is the first CC, created by the anonymous person(s) named Satoshi Nakomoto in 2009 and has a limited supply of 21 million coins. It uses the SHA-256 Proof-of-Work hashing algorithm.
2	Ethereum	ETH	ETH is a Turing-complete CC platform created by Vitalik Buterin. It raised US$18 million worth of bitcoins during a crowdsale of ether tokens in 2014.
3	Bitcoin cash	BCH	BCH brings a reliable currency to the world, fulfilling the original bitcoin's promise of "point-to-point digital cash" and provides businesses and users with low transaction fees and reliable transaction confirmation.
4	Ripple	XRP	Ripple is a payment system created by Ripple Labs in San Francisco. It allows for banks worldwide to transact with each other without the need of a central correspondent. It was one of the earliest altcoin in the market and is not a copy of BTC's source code.

5	Litecoin	LTC	Litecoin is branded the "silver to bitcoin's gold". It was created by Charles Lee, an ex-employee of Google and current employee of Coinbase.
6	IOTA	IOT	IOT, an open-source distributed ledger protocol, goes "beyond blockchain" through its core invention of the blockless Tangle. The IOTA Tangle is a quantum-proof Directed Acyclic Graph, with no fees on transactions and no fixed limit on how many transactions can be confirmed per second in the network.
7	Cardano	ADA	ADA, Designed and developed by IOHK, innovates Cardano Settlement Layer as a Proof of Stake cryptocurrency based on the Haskell implementation of the white paper Ouroboros: A Provably Secure Proof of Stake Blockchain Protocol.
8	Dash	DASH	Dash is a privacy-centric cryptocurrency. It anonymizes transactions using PrivateSend, a concept that extends the idea of CoinJoin.
9	NEM	XEM	XEM, a CC platform launched in 2015, is written from scratch on the Java platform. It provides many services on top of payments such as messaging, asset making and naming system.
10	Monero	XMR	XMR is the leading cryptocurrency with a focus on private and censorship-resistant transactions

Table 8: Top 10 CCs used in construction of CRIX

Table 9: Option price for CRIX calls at different time to maturity τ and strike prices K for SVJ. 20170717

$K\backslash\tau$	1.00	7.00	30.00	60.00	90.00	180.00	360.00	720.00
5500.00	1519.23	1653.64	1948.01	2213.13	2236.85	2741.01	3043.31	3432.85
5600.00	1419.07	1614.03	1856.26	2198.38	2244.50	2647.72	3052.62	3468.09
5700.00	1364.69	1528.17	1833.97	2067.55	2219.64	2561.57	2877.45	3413.68
5800.00	1301.39	1504.09	1814.05	2061.48	2165.44	2554.22	2943.87	3442.18
5900.00	1238.29	1461.01	1723.13	2060.90	2155.57	2468.25	2926.67	3428.88
6000.00	1171.25	1422.40	1719.12	1876.69	2139.17	2401.04	2766.27	3232.10
6100.00	1112.21	1341.72	1662.11	1933.53	2114.46	2445.15	2787.49	3181.24
6200.00	1075.23	1341.71	1622.08	1901.96	2006.56	2476.04	2929.48	3384.33
6300.00	1033.49	1259.66	1554.50	1843.90	2118.94	2425.29	2777.42	3405.77
6400.00	907.65	1240.67	1572.87	1827.40	1973.01	2355.75	2654.88	3198.64
6500.00	893.65	1167.71	1533.27	1745.53	1962.42	2383.25	2725.50	3338.71
6600.00	820.08	1161.17	1484.08	1742.47	1967.27	2288.48	2747.74	3052.27
6700.00	793.46	1083.36	1492.16	1703.92	1882.94	2321.05	2616.79	3164.87
6800.00	813.15	1031.33	1380.94	1714.34	1808.57	2199.88	2733.80	3001.09
6900.00	716.85	1008.18	1373.28	1616.95	1797.88	2210.80	2646.95	3199.27
7000.00	691.51	1011.13	1327.54	1580.70	1859.21	2155.43	2582.22	2971.25
7100.00	623.76	898.85	1299.75	1532.40	1727.76	2153.71	2548.25	2926.45
7200.00	589.95	896.01	1269.16	1574.41	1731.65	2150.75	2430.03	3108.01
7300.00	549.96	851.58	1247.41	1537.25	1727.48	2027.80	2563.57	3082.29
7400.00	585.51	860.62	1223.00	1517.07	1615.95	2055.82	2460.49	2954.29
7500.00	484.38	785.55	1182.36	1378.83	1592.91	2017.55	2361.38	2983.25

5.1 Additional Tables for Pricing

Table 10: Option price for CRIX calls at different time to maturity τ and strike prices K for SV. 20170717

$K\backslash\tau$	1.00	7.00	30.00	60.00	90.00	180.00	360.00	720.00
5500.00	1278.72	1559.09	1959.85	2136.31	2291.23	2627.61	2996.95	3223.16
5600.00	1190.57	1512.70	1885.56	2102.15	2263.43	2523.04	3011.67	3203.38
5700.00	1118.34	1437.43	1732.72	2040.55	2292.38	2503.40	3046.01	3180.93
5800.00	1044.16	1389.84	1717.11	1981.39	2166.67	2488.79	2876.14	3173.82
5900.00	985.43	1250.46	1647.45	1970.84	1975.05	2358.53	2840.98	3365.54
6000.00	929.09	1272.21	1625.65	1957.72	2075.81	2370.56	2689.30	3127.72
6100.00	846.08	1221.00	1579.72	1887.34	1974.22	2331.31	2745.73	3283.08
6200.00	787.51	1152.98	1559.45	1846.29	2022.94	2304.82	2681.89	3030.52
6300.00	736.92	1099.30	1542.18	1721.37	1984.72	2157.44	2773.65	3082.67
6400.00	683.88	1096.25	1447.81	1706.58	1923.64	2151.32	2624.78	3117.61
6500.00	629.67	1003.69	1421.64	1726.32	1843.63	2290.61	2604.47	3067.25
6600.00	570.72	1002.77	1402.49	1621.71	1850.89	2091.57	2538.98	2976.26
6700.00	536.96	922.66	1417.76	1577.39	1804.97	2122.91	2596.53	3124.71
6800.00	479.85	885.41	1309.41	1620.85	1753.63	2081.38	2687.52	2938.64
6900.00	443.59	841.91	1257.76	1529.91	1731.73	1935.59	2365.98	2958.66
7000.00	397.46	817.77	1295.92	1506.96	1703.15	2096.44	2440.30	2994.91
7100.00	358.85	779.26	1215.88	1491.60	1735.59	1997.29	2306.17	3007.89
7200.00	346.01	718.90	1187.23	1484.38	1716.16	1972.58	2284.85	2735.76
7300.00	297.17	707.24	1128.35	1411.96	1592.25	1904.08	2430.86	2766.59
7400.00	280.01	688.76	1156.22	1388.58	1621.48	1845.45	2426.65	2891.36
7500.00	247.22	635.55	1075.68	1357.45	1519.07	1944.85	2284.36	2915.14

Table 11: Option price for BTC calls at different time to maturity τ and strike prices K for SVJ. 20170717

$K\backslash\tau$	1.00	7.00	30.00	60.00	90.00	180.00	360.00	720.00
1250.00	1064.85	1033.01	1100.20	1123.24	1160.75	1201.23	1327.60	1472.02
1350.00	959.09	970.52	1026.62	1061.08	1084.23	1192.36	1369.41	1487.17
1450.00	886.93	867.61	954.97	1016.83	1071.43	1154.19	1255.90	1459.35
1550.00	809.50	804.15	889.15	961.11	1005.34	1063.62	1258.91	1283.68
1650.00	721.23	768.59	826.53	898.89	963.17	1028.24	1215.46	1427.56
1750.00	664.76	694.54	787.95	851.09	944.14	1064.60	1214.10	1249.38
1850.00	570.52	635.94	711.84	852.07	885.28	1017.04	1121.49	1304.71
1950.00	512.29	576.61	681.92	796.60	834.36	982.20	1114.49	1176.52
2050.00	427.11	520.25	641.50	739.36	772.18	941.37	1125.87	1265.28
2150.00	380.08	469.71	603.82	685.08	737.93	916.16	1099.15	1253.77
2250.00	326.57	438.30	590.85	666.83	758.94	837.28	997.85	1255.53
2350.00	269.49	396.02	537.18	626.14	691.93	832.70	1031.18	1083.78
2450.00	237.23	370.24	511.91	607.20	666.93	820.12	978.48	1255.60
2550.00	204.33	332.11	448.49	562.52	639.45	786.32	893.43	1193.56
2650.00	172.57	302.61	451.29	552.30	617.59	747.86	847.73	1162.78
2750.00	144.18	273.33	432.72	542.46	625.52	725.66	954.69	1130.77
2850.00	123.83	253.31	398.65	489.49	569.56	714.75	837.14	1051.28
2950.00	107.31	210.62	368.57	489.35	539.88	700.19	900.59	1052.99
3050.00	93.15	191.68	357.75	461.92	546.27	678.59	830.83	1006.12
3150.00	81.37	187.33	326.54	423.21	503.43	644.42	869.81	1071.14
3250.00	81.85	167.41	313.05	401.94	487.19	641.72	889.76	985.55

Table 12: Option price for BTC calls at different time to maturity τ and strike prices K for SV. 20170717

$K\backslash\tau$	1.00	7.00	30.00	60.00	90.00	180.00	360.00	720.00
1250.00	995.84	1004.22	1029.74	1082.74	1084.83	1179.59	1216.10	1346.55
1350.00	903.13	917.99	953.02	1001.62	1037.17	1104.18	1143.62	1328.01
1450.00	797.56	826.45	911.42	927.01	971.60	1056.94	1115.92	1211.31
1550.00	714.15	748.50	811.92	872.76	924.94	984.84	1089.48	1195.81
1650.00	614.00	671.71	757.16	800.37	864.46	930.87	1048.44	1165.70
1750.00	530.73	598.15	691.99	743.59	809.24	873.12	989.64	1151.55
1850.00	450.97	528.07	634.70	711.05	750.49	804.50	912.36	1103.70
1950.00	372.29	462.11	567.54	683.99	703.14	787.34	871.95	1053.84
2050.00	300.44	404.72	544.71	597.14	661.98	748.35	875.81	1007.02
2150.00	240.73	356.16	488.93	561.01	611.53	704.67	828.16	980.96
2250.00	186.15	305.61	440.95	538.28	559.86	697.84	743.97	965.96
2350.00	142.91	273.56	398.06	489.53	548.33	629.77	789.06	932.60
2450.00	101.17	225.25	362.18	462.04	520.66	622.08	744.72	917.09
2550.00	73.96	200.60	334.88	414.82	491.67	585.88	713.12	850.85
2650.00	51.01	170.79	320.81	378.28	447.84	535.59	661.32	856.58
2750.00	35.60	145.28	261.67	356.89	411.18	523.39	660.04	861.44
2850.00	23.81	123.14	261.85	331.88	413.75	506.06	635.91	777.71
2950.00	14.54	100.39	231.20	300.13	365.73	473.96	665.24	823.04
3050.00	9.25	85.84	211.53	290.34	367.98	441.68	619.74	797.52
3150.00	5.05	78.52	190.86	277.61	326.19	412.51	530.80	767.74
3250.00	2.65	65.87	178.61	260.99	317.15	415.77	563.82	681.10

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